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Abstract

An exchange economy is said to be in equilibrium if at some price vector the demand for each commodity equals its supply. The corresponding price vector is called an equilibrium price vector. The purpose of this paper is to introduce a new price adjustment process which is globally convergent. The latter means that the price adjustment process may start at an arbitrary price vector and terminates with an equilibrium price vector for arbitrary exchange economies. The price adjustment process is based on the following two informational inputs. For each price vector, its position relative to the starting price vector needs to be known, and the aggregate excess demand needs to be evaluated.

Under the price adjustment process only the prices of commodities having the highest excess demand are increased from their initial levels. Simultaneously, only the prices of commodities having the highest excess supply, i.e., lowest excess demand, are decreased from their initial levels. In this manner, the demand of a commodity with lowest excess demand is typically, but not necessarily, increased, while the demand for a commodity with highest excess demand is typically decreased. To ensure convergence of the price adjustment process, it is necessary to keep a price which has been raised or has been lowered from its starting level, fixed at this initial level as soon as during the process that price becomes equal to its initial price again.

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1 Introduction

At an economic equilibrium, the demand for each commodity is, given the prices for all commodities, precisely equal to the supply of that commodity. The prices of the commodities play an important role for the levels of the demand as well as the supply on the market of each commodity. The market for a commodity is said to be in equilibrium if the excess demand for that commodity is equal to zero. An equilibrium price vector is a price vector satisfying that the markets of all commodities are in equilibrium. The question if and how an economic equilibrium can be reached starting from a situation where the economy is not in equilibrium, has attracted much theoretical attention. Given the crucial role of the prices of the commodities in the economy, it is hardly surprising that many contributions in the literature have focussed on price adjustment processes.

An informal model in an exchange framework was proposed already by Walras (1874) as a solution to the problem of reaching an equilibrium price vector. It is, however, not difficult to construct an economy for which this so-called successive tâtonnement process does not converge to an equilibrium price vector, meaning that it continues forever without reaching an equilibrium price vector.

Samuelson (1947) formulated an improvement of Walras' successive tâtonnement process, the so-called simultaneous tâtonnement process, as a set of differential equations which formalize that the price of each commodity changes proportional to its excess demand at any point in time. As the name indicates, all prices are changed simulaneously throughout this process, hence information available from the markets of all commodities is used. For some time, scientific interest focussed on finding conditions to ensure the convergence of Samuelson's simultaneous tâtonnement process and related price adjustment processes, see e.g., Arrow and Hurwicz (1958), Arrow *et al.* (1959), Uzawa (1961). The conditions provided by the latter contributions on the excess demand function and the price adjustment processes, which are necessary to assure convergence of the dynamical process to an equilibrium price vector, are very strong.

Scarf (1960) demonstrated that any project of finding (realistic) necessary and sufficient conditions for convergence of such price adjustment processes was bound to be too ambitious. For an exchange economy with one equilibrium price vector, Scarf showed that the simultaneous tâtonnement process may cycle forever without reaching an equilibrium price vector. Furthermore, the contributions of Sonnenschein (1972, 1973), Mantel (1974), and Debreu (1974) showed definitely and explicitly that any function fulfilling only three conditions, namely continuity, Walras' law, and homogeneity of degree zero in prices, can be an excess demand function for an exchange economy. Hence, even for the highly stylized setting of an exchange economy, almost any dynamical process should be anticipated as the result of combining Samuelson's price adjustment process with some excess demand function. Therefore, the conditions on the excess demand functions ensuring convergence to equilibrium found until then, must be viewed as unduely strong. A very promising alternative price adjustment process was proposed by Smale (1976). This process is known to converge to an equilibrium price vector for arbitrary excess demand functions. However, it must be started on the boundary of the price space to ensure convergence to an equilibrium price vector, which makes the economic interpretation of the path of prices generated by this process somewhat cumbersome and its applicability limited.

Scarf (1967) and Kuhn (1968) formulated two different types of processes for computing an equilibrium price vector. The sequence of price vectors generated by each of these computational processes, forms a path of points in the unit simplex, being the set of nonnegative price vectors for which the components add up to unity. This path of price vectors may be interpreted as a price adjustment process. Scarf's computational process is the dual of Kuhn's process in the following sense. For the former process, initially the price of the commodity which has the lowest excess demand is decreased, and a path of points in the price space is generated satisfying the property that the excess demands for all other commodities are kept approximately equal to each other. For Kuhn's process, however, the price of the commodity which has the highest excess demand is increased initially, and the prices of all other commodities are changed such that their excess demands are kept approximately equal to each other. Both processes terminate at an equilibrium price vector for arbitrary exchange economies provided all goods are desirable. To ensure convergence, however, these processes must be started at a price vector, where one or more commodities have prices are equal to zero. An attractive economic interpretation of these price adjustment processes is therefore not straightforward.

The price adjustment process which we present in this paper, may be regarded as a synthesis between the processes of Scarf and Kuhn. In contrast to the latter two processes however, the process may start at an arbitrary price vector in the unit simplex. In the process, the price of the commodity having the highest excess demand is initially increased, and simultaneously the price of the commodity having the highest excess supply, i.e., the lowest excess demand, is decreased. The relative prices of the other commodities are initially not changed. Throughout the entire process of price adjustment, the prices of commodities with the highest excess demand are allowed to be higher than their initial values as in Scarf's process, while the prices of commodities with the lowest excess demand are allowed to be lower than their initial values, as in Kuhn's process. The prices of the commodities which have neither maximal nor minimal excess demand, remain equal to their initial values.

Other processes that may start at any price vector were introduced by van der Laan and Talman (1987b) and Kamiya (1990). In van der Laan and Talman (1987b) an adjustment process is proposed in which initially the price of each commodity for which the excess demand is positive (negative), is increased (decreased) proportionally to its level at the starting price vector. In general, this process keeps the prices of the commodities with positive (negative) excess demand relatively maximal (minimal). In the process of Kamiya (1990) a homotopy function is introduced which deforms a trivial system having a unique solution, to the system of the excess demand function.

One attractive feature of our price adjustment process is its natural interpretation, namely the relative prices of the commodities which are the scarcest and the most abundant, are changed first. Actually, raising the prices of the scarcest commodities and lowering the prices of the most abundant ones, makes sense economically. In this way, in general the excess demand (supply) of a commodity having the highest excess demand (supply) decreases. Moreover, the path of relative prices for a pure exchange economy with $n+1$ commodities followed by the price adjustment process may be approximated by the so-called $n(n+1)$-ray variable dimension restart algorithm of Joosten and Talman (1993), which was designed for computing economic equilibria on the n-dimensional unit simplex.

The paper is organized as follows. In the following section, we discuss computational processes, price adjustment processes, and convergence properties of these processes rather informally. In Section 3, we formulate the underlying model of an exchange economy and several other concepts relevant for a description of the price adjustment process. We furthermore show the existence of an equilibrium price vector for arbitrary aggregate excess demand functions. In Section 4, we formulate the globally convergent price adjustment process. We demonstrate how the price adjustment process works by means of a detailed description for an example.

2 Price adjustment processes and criteria for convergence

In Walras' successive tâtonnement process, the 'auctioneer' ignores all information from the markets of other commodities, while adjusting the price of a certain commodity in order to achieve an equilibrium on that market. As an improvement, Samuelson (1947) formulated the so-called simultaneous tâtonnement process where all prices are changed simultaneously, as follows

$$\frac{dp}{dt} = z(p), \tag{1}$$

where for an economy with commodities indexed $j \in I^{n+1} = \{1, ..., n+1\}$, the vector $z(p) = (z_1(p), ..., z_{n+1}(p))^\top$ is the excess demand evaluated at price (vector) $p = (p_1, ..., p_{n+1}) \in R^{n+1}_+ \setminus \{0^{n+1}\}$, where $R^{n+1}_+ = \{x \in R^{n+1} \mid x_j \geq 0$ for all $j \in I^{n+1}\}$, and 0^{n+1} is the $(n+1)$-vector of zeroes. If the function z is continuous (Lipschitz continuous or, more strongly, continuously differentiable), there exists a (unique) solution curve $\{p(t)\}_{t\geq 0}$ satisfying $p(0) = p_0 \in R^{n+1}_+ \setminus \{0^{n+1}\}$ and Equation (1). By using Walras' law, i.e., $p^\top z(p) = 0$ for all $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$, it may be confirmed that on the trajectory $\{p(t)\}_{t\geq 0}$ the length (measured in the Euclidean norm $\| \, . \, \|_2$) of the vectors being generated remains constant. Indeed for $p = p(t)$, it follows that $d(\| p \|_2^2)/dt = d(\sum_{j\in I^{n+1}} p_j^2)/dt = \sum_{j\in I^{n+1}} 2p_j dp_j/dt = 2\sum_{j\in I^{n+1}} p_j z_j(p) = 0$ by Walras' law.

The interpretation of the simultaneous tâtonnement process is intuitively and economically appealing. If the excess demand of a good is positive (negative), then its price is increased (decreased) proportional to its excess demand. We refer to Arrow and Hurwicz (1958), Arrow *et al.* (1959), Uzawa (1961) for sufficient conditions under which a large class of price adjustment processes, to which Samuelson's tâtonnement process belongs, converges. These conditions are generally viewed as quite strong, and certainly not all economies satisfy these conditions. For some time attention in the profession focused on finding weaker conditions on excess demand functions and price adjustment processes for which convergence to an equilibrium price vector was guaranteed. Among these conditions, conditions for uniqueness of an equilibrium price vector received particular interest. However, Scarf

(1960) showed conclusively that the simultaneous tâtonnement process may not converge to an equilibrium price vector, even if this equilibrium price vector is uniquely determined.

The results of Sonnenschein (1972, 1973), Mantel (1974), and Debreu (1974) imply that an excess demand function for an exchange economy is characterized by only three properties, namely continuity, Walras' law, and homogeneity of degree zero in the prices. Under very weak conditions, every exchange economy with utility maximizing agents, yields an aggregate excess demand function fulfilling these three conditions. Moreover, for any continuous function satisfying Walras' law and homogeneity of degree zero in the prices, an exchange economy with utility maximizing agents can be constructed, such that the resulting aggregate excess demand function is equal to the former function arbitrarily closely on an arbitrarily large subset of the interior of the space of all possible price vectors.

Smale (1976) formulated a more sophisticated method of price adjustment, called the Global Newton method. This method has the form, for $t \geq 0$,

$$\begin{aligned} p_{n+1}(t) &= 1, \text{ and} \\ z_j(p(t)) &= \lambda(t) z_j(p(0)),\ j = 1, ..., n \end{aligned} \tag{2}$$

where z is a continuously differentiable excess demand function. The scalar $\lambda(t)$ is a real valued function evaluated at point t in time. For almost all initial price vectors on the boundary of $R_+^{n+1} \backslash \{0^{n+1}\}$ with $p_{n+1} = 1$, the Global Newton method converges to an equilibrium price vector, provided the eigenvalues of the $(n \times n)$-Jacobian-matrix $Dz(p)$ of the function $(z_1, ..., z_n)$ are all nonzero at a zero of z.

Equations (2) can be rewritten as

$$\frac{dz_j(p)}{dt} = -\lambda(t) z_j(p),\ j = 1, ..., n. \tag{3}$$

Equation (3) implies that under this price adjustment process the change in the excess demand of each of the first n commodities, is proportional to the excess demand itself. Under Scarf's computational process (1967), when started at the vertex $e_{n+1} = (0, ..., 0, 1)^\top$ of the n-dimensional unit simplex

$$S^n = \{x \in R_+^{n+1} \mid x_j \geq 0 \text{ for all } j \in I^{n+1}, \text{ and } \sum_{j=1}^{n+1} p_j = 1\}, \tag{4}$$

a path of prices is followed satisfying

$$z_j(p) = z_k(p) \text{ for all } j, k \in I^{n+1}, \text{ such that } j, k \neq n+1,$$

for all prices p on the path. Hence, the changes in the first n components of the excess demand function z are proportional to z itself in this process as well. Several authors have remarked that Smale's Global Newton method follows the same path as Scarf's computational process when the former is started at the price vector $(0, ..., 0, 1)^\top$. An alternative manner of describing the path of points p followed by the processes of Scarf and Smale when started at $v = (0, ..., 0, 1)^\top$, is

$$\begin{aligned} p_j &\geq v_j \text{ and } z_j(p) = \max_{h \in I^{n+1}} z_h(p), \; j = 1, ..., n, \\ p_{n+1} &\leq v_{n+1} \text{ and } z_{n+1}(p) = \min_{h \in I^{n+1}} z_h(p). \end{aligned}$$

A price adjustment process is said to be *universally convergent* if it converges to an equilibrium price vector for (almost) all aggregate excess demand functions. If the price adjustment process additionally accomplishes this feat while being started in an (almost) arbitrary price vector, then it is called *globally convergent*. We call a process *effective* (in the sense of Saari and Simon, 1978), if the solutions converge to an equilibrium price vector for (almost) all initial price vectors in some subset of the space on which the excess demand is given. Samuelson's tâtonnement process is not universally convergent, which follows from Scarf (1960), hence it is also not globally convergent. The related computational processes of Scarf (1967, 1973) and Kuhn (1968, 1969) must start on the boundary of the price space, hence while being universally convergent, these methods are not globally convergent. Smale's Global Newton method is both universally convergent and effective, but it is not globally convergent, since it must start on the boundary of the price space to ensure convergence. Keenan (1981) also demonstrated that the Global Newton method is also *locally effective* in the sense that convergence to an equilibrium price vector is guaranteed if the method is started near an equilibrium price vector, albeit not necessarily to the same equilibrium price vector. Keenan (1981) has shown that for a completely arbitrary starting point, convergence of Smale's method is not guaranteed.

Recently, several contributions have appeared in the literature each linking a certain globally convergent price adjustment process to a so-called variable dimension restart algorithm. The class of variable dimension restart algorithms for finding an equilibrium price vector originated with the contribution of van der Laan and Talman (1979). This algorithm generalized Scarf's algorithm (1973) in the sense that the process of van der Laan and Talman may start at an arbitrary price vector in the interior of the state space, which is the n-dimensional unit simplex in case the economy has $n+1$ commodities. The first round of the algorithm of van der Laan and Talman may start with a rather large step size, and this round terminates rather quickly with a first approximation of an equilibrium price vector. If the excess demand at this price vector does not fulfil a given accuracy criterion, the procedure is restarted near the approximating equilibrium price vector found in the first round, with a smaller step size. This second round terminates with another approximation of the equilibrium price vector, and in general the excess demand at this price vector is closer to zero in each component than the approximation found in the first round. Each round terminates in a finite number of steps with an approximation of an equilibrium price vector. Whenever the excess demand function at this approximation does not fulfil the given accuracy criterion, the procedure is restarted with a smaller step size. In this manner, an approximation of an equilibrium price vector can be found such that the excess demand function evaluated at this price vector, fulfils the arbitrary predetermined accuracy criterion.

Several improvements and generalizations of the original variable dimension restart algorithm of van der Laan and Talman (1979) have been developed since, e.g., Doup and Talman (1987), van der Laan *et al.* (1987) and Doup *et al.* (1987). By these contributions, considerable gains in computational efficiency relative to the efficiency of the algorithms of Scarf (1967, 1973) and Kuhn (1968, 1969), but also relative to the efficiency of the original algorithm of van der Laan and Talman (1979), have been accomplished. Furthermore, several of these variable dimension restart algorithms generate paths of prices which allow for more natural interpretations as economically plausible paths of a price adjustment process (see, e.g., van der Laan and Talman, 1987a, and van den Elzen, 1993). The price adjustment process which we are about to present in this paper, is closely related to a variable dimension restart algorithm as well, namely the $n(n+1)$-ray algorithm for finding an equilibrium price vector on the unit simplex, proposed by Joosten and Talman (1993).

3 Model and preliminaries

We consider a competitive exchange economy with $n+1$ commodities, indexed $j \in I^{n+1} = \{1, ..., n+1\}$. The set of all possible price vectors is $R^{n+1}_+ \setminus \{0^{n+1}\}$, where 0^{n+1} is the $(n+1)$-vector consisting of zeros. Suppose that the economy has m consumers. Each consumer is characterized by his consumption set, initial endowments , and preference relation. For each consumer $i \in I^m = \{1, ..., m\}$ the following holds.

a) The consumption set X^i is a compact, convex subset of R^{n+1}_+, containing the set

$$\{x \in R^{n+1} \mid 0 \leq x_j \leq \sum_{h=1}^{m} w^h_j + 1, \text{ for all } j \in I^{n+1}\},$$

where $w^h = (w^h_1, ..., w^h_{n+1})^\top$ is the vector of initial endowments of the $n+1$ commodities of consumer $h \in I^m$;

b) $w^i_j > 0$ for all $j \in I^{n+1}$;

c) The preference relation $\succeq_i$ is continuous, monotonic and strictly convex.

Let $B^i(p) = \{x \in X^i \mid p^\top x \leq p^\top w^i\}$ be the budget set of consumer $i \in I^m$ given price vector $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$. We assume that each consumer $i \in I^m$ maximizes his utility over his budget set, i.e., he chooses a maximal element with respect to his preferences $\succeq_i$ in the budget set $B^i(p)$. Under a)-c), for every $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$ this utility maximizing element is unique, moreover it is a boundary element of the budget set. Let $x^i(p)$ be the utility maximizing demand of consumer i given the price $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$, then $p^\top x^i(p) = p^\top w^i$, for each consumer $i \in I^m$. Furthermore, $x^i : R^{n+1}_+ \setminus \{0^{n+1}\} \to R^{n+1}$ is continuous for each consumer $i \in I^m$.

Let $z^i : R^{n+1}_+ \setminus \{0^{n+1}\} \to R^{n+1}$, be the excess demand function of consumer $i \in I^m$, defined by $z^i(p) = x^i(p) - w^i$ for every $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$. Then, $p^\top z^i(p) = 0$ for every $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$. The aggregate excess demand function, defined by $z(p) = \sum_{i \in I^m} z^i(p)$ for every $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$, is a continuous function satisfying:

i) $p^\top z(p) = 0$ for all $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$ (Walras' law),

ii) $z_j(p) > 0$ whenever $p_j = 0$ (desirability),

iii) $z(\mu \cdot p) = z(p)$ for all scalars $\mu > 0$ and all $p \in R^{n+1}_+ \setminus \{0^{n+1}\}$ (homogeneity of degree zero in prices).

Note that the aggregate excess demand of each commodity depends not only on its own price, but also on the prices of all other commodities. Sonnenschein (1972, 1973), Mantel (1974), and Debreu (1974) showed furthermore that these three properties characterize aggregate excess demand functions. The last property of the excess demand function, i.e., homogeneity of degree zero in prices, allows normalization of the price space to the n-dimensional unit simplex, for instance, or setting one price equal to one (numéraire), or normalization to the intersection of the $(n+1)$-dimensional unit ball and R_+^{n+1}. For any such normalization property (iii) obviously becomes void. The main advantage of the n-dimensional unit simplex is that this set is both convex and compact, whereas $R_+^{n+1}\backslash\{0^{n+1}\}$ is neither bounded, nor closed. The intersection of the $(n+1)$-dimensional unit ball and R_+^{n+1} is compact, but not convex, whereas taking a numéraire good gives a set not being bounded. In the remainder, we restrict our analysis of excess demand functions to the normalized price space formed by the unit simplex. For an illustration of the n-dimensional unit simplex for $n=2$, we refer to Figure 1. In the following proposition, we state the well-known fact that an equilibrium price vector always exists on the n-dimensional unit simplex for any aggregate excess demand function.

Proposition 1 *Let $z : S^n \to R^{n+1}$ be an aggregate excess demand function. Then, there exists $p^* \in S^n$ satisfying $z(p^*) = 0^{n+1}$.*

Existence of an equilibrium price vector for an arbitrary excess demand function is therefore guaranteed. However, as we have seen in the previous section, existence of an economically plausible globally convergent price adjustment process is by no means straightforward. In the following section, we present such a price adjustment process, and in order to facilitate the exposure somewhat, we introduce several notations here. We denote the arbitrary starting point of the price adjustment process by $v \in S^n$. For $A \subset I^{n+1}$, $-A$ is defined to be the set $\{-j \mid j \in A\}$. Furthermore, let T be a subset of $I^{n+1} \cup -I^{n+1}$, satisfying $T \cap I^{n+1} \neq \emptyset$, $T \cap -I^{n+1} \neq \emptyset$, $(T \cap -I^{n+1}) \subset \{-j \mid v_j > 0\}$, and $T \cap -T = \emptyset$. Let $T^+ = \{j \in I^{n+1} \mid j \in T\}$, $T^- = \{j \in I^{n+1} \mid -j \in T\}$, and $T(c) = \{h \in I^{n+1} \mid h \notin T \cup -T\}$.

Definition 2 *Let $v \in S^n$ be given. Let $T \subset I^{n+1} \cup -I^{n+1}$ be as described, then the subset $A(T)$ of S^n is given by*

$$A(T) = \{p \in S^n \mid p_j \leq v_j \text{ if } j \in T^-,\ p_j = v_j \text{ if } j \in T(c),\ p_j \geq v_j \text{ if } j \in T^+\}.$$

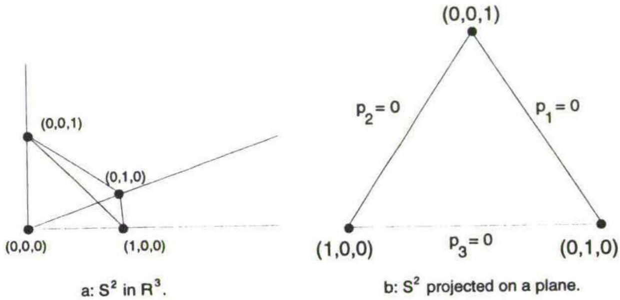

Figure 1: The 2-dimensional unit simplex.

The dimension of $A(T)$ is equal to $| T | - 1$. Hence, $1 \leq \dim A(T) \leq n$ for all T as described above. Furthermore, the union of the sets $A(T)$ over all $T \subset I^{n+1} \cup -I^{n+1}$ with $T(c) = \emptyset$, is the unit simplex. The interpretation of such a set $A(T)$ is straightforward. Any point in the interior of the set $A(T)$ may be reached from the starting point v by increasing the prices of the commodities (of which the index corresponds with an index) in T^+, decreasing the prices of the commodities in T^-, while keeping the prices of the commodities in $T(c)$ at the initial level. For an illustration, see Figure 2.

Definition 3 *Given $T \subset I^{n+1} \cup -I^{n+1}$ as described, then the subset $C(T)$ of S^n is given by*

$$C(T) = \{p \in S^n \mid z_j(p) = \min_{h \in I^{n+1}} z_h(p) \text{ if } j \in T^-,$$
$$\min_{h \in I^{n+1}} z_h(p) \leq z_j(p) \leq \max_{h \in I^{n+1}} z_h(p) \text{ if } j \in T(c),$$
$$z_j(p)) = \max_{h \in I^{n+1}} z_h(p) \text{ if } j \in T^+\}.$$

For an illustration, we refer to Figure 3. For the sake of comparison we have depicted the same point v as in Figure 2. Since $v \in C(\{+3, -1\})$,

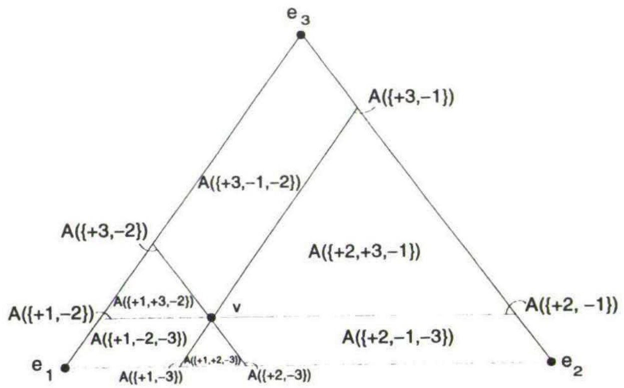

Figure 2: The subdivision of the 2-dimensional unit simplex into sets A(T) of varying dimension.

it follows at the price vector v that the excess demand of commodity 3 is maximal and the excess demand of commodity 1 is minimal. Now, we derive two lemmas, which will be useful in the next section.

Lemma 4 *Let $z : S^n \to R^{n+1}$ be an excess demand function. Let $p^* \in S^n$ satisfy $z_j(p^*) = \beta$ for all $j \in I^{n+1}$. Then p^* is an equilibrium price vector.*

Proof. By Walras' law, $0 = \sum_{j \in I^{n+1}} p_j^* z_j(p^*) = \sum_{j \in I^{n+1}} p_j^* \beta = \beta$. Hence, $z(p^*) = 0^{n+1}$. ■

Lemma 5 *Let $p^* \in C(T)$, then $p_j^* > 0$ for all $j \in T^-$.*

Proof. By desirability, $p_j^* = 0$ implies $z_j(p^*) > 0$, whereas $j \in T^-$ implies $z_j(p^*) = \min_{h \in I^{n+1}} z_h(p^*)$. Suppose $p_j^* = 0$ for some $j \in T^-$. Then, $\min_{h \in I^{n+1}} z_h(p^*) = z_j(p^*) > 0$. Hence,

$$0 = \sum_{k \in I^{n+1}} p_k^* z_k(p^*) \geq \min_{h \in I^{n+1}} z_h(p^*) > 0,$$

which yields a contradiction. ■

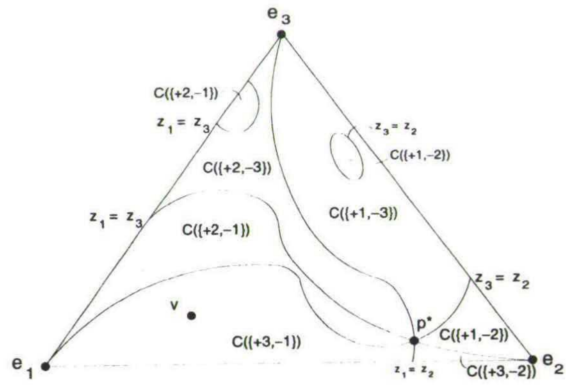

Figure 3: A subdivision of the 2-dimensional unit simplex into sets C(T). The curves indicate sets of price vectors where two commodities have equal excess demand. The point p*, where the excess demands of all commodities are equal, is the unique equilibrium price vector.

4 The globally convergent price adjustment process

The price adjustment process may start at an arbitrary point of the unit simplex, representing a vector of normalized relative prices. At the starting point $v \in S^n$, following Herings (1994) we may assume that there is precisely one good, say i^*, with maximal positive aggregate excess demand, and there is precisely one other good, say $j^* \in Car(v) = \{j \in I^{n+1} \mid v_j > 0\}$, with minimal negative aggregate excess demand. Hence, $v \in C(\{+i^*, -j^*\})$. Then, under the price adjustment process, initially p_{i^*}, the price of commodity i^*, is increased from v_{i^*}, and simultaneously p_{j^*}, the price of commodity j^*, is decreased from v_{j^*}. The prices of all other commodities remain unchanged initially.

Throughout the entire process, we make the following nondegeneracy assumption. At any point in S^n reached by the price adjustment process which is not an equilibrium price vector, it holds that for at most one commodity the excess demand becomes maximal or minimal, or its price becomes equal to the initial value again. This nondegeneracy assumption is made for convenience in the ensuing description of the price adjustment process, and it holds in general.

The price adjustment process leaves the starting point v, along the ray formed by the one-dimensional set $A(\{+i^*, -j^*\})$ by increasing the price of good i^* from the initial level v_{i^*}, decreasing the price of good j^* from the initial level v_{j^*}, and keeping the prices of all other commodities fixed at their initial levels. One of the following three mutually exclusive situations will occur along $A(\{+i^*, -j^*\})$:

1) For some price vector p^* it holds that $p^*_{j^*} = 0$;

2) For some price vector p^* it holds that the excess demand of some commodity, say $i' \in T(c)$, becomes equal to the excess demand of commodity i^*;

3) For some price vector p^* it holds that the excess demand of some commodity, say $j' \in T(c) \cap Car(v)$, becomes equal to the excess demand of commodity j^*.

Situation 1 cannot occur according to Lemma 5.

If Situation 2 occurs, then the excess demand of commodity i' becomes maximal. The process continues by raising the price of commodity i', from its initial level $v_{i'}$, and by keeping z_{i^*} and $z_{i'}$, the excess demands of commodities i^* and i', respectively, equal to each other. Thus, the set $A(\{+i^*, +i', -j^*\})$ of S^n is entered and a path of points in this set is followed satisfying $z_{i^*}(p) = z_{i'}(p)$ for each point p of the path.

If Situation 3 occurs, then the excess demand of commodity j' becomes minimal. The process continues by lowering the price of commodity j', from its initial level $v_{j'}$, and by keeping z_{j^*} and $z_{j'}$, the excess demands of commodities j^* and j', respectively, equal to each other. Thus, the set $A(\{+i^*, -j^*, -j'\})$ of S^n is entered and a path of points in this set is followed satisfying $z_{j^*}(p) = z_{j'}(p)$ for each point p of the path.

In general, the price adjustment process traces a path in some set $A(T)$ for varying T satisfying for every p on a curve in $A(T)$ that

$$p_j \geq v_j \text{ and } z_j(p) = \max_{h \in I^{n+1}} z_h(p) \text{ for all } j \in T^+,$$

$$p_j = v_j \text{ and } \min_{h \in I^{n+1}} z_h(p) \leq z_j(p) \leq \max_{h \in I^{n+1}} z_h(p) \text{ for all } j \in T(c), \tag{5}$$

$$p_j \leq v_j \text{ and } z_j(p) = \min_{h \in I^{n+1}} z_h(p) \text{ for all } j \in T^- .$$

Generically, following the proof of Theorem 10.4.2 of Herings (1995), the set of points satisfying (5) for some set T consists of a finite number of curves, each curve being a loop or a path with two end points. Then, one of the following mutually exclusive cases may occur at an end point of a path in $A(T)$:

1) The price adjustment process reaches a point $p^* \in bd(A(T))$;

2) The price adjustment process reaches a point $p^* \in int(A(T))$ such that for some $j^* \in T(c)$ it holds that $z_{j^*}(p^*)$ either becomes maximal or becomes minimal;

3) The price adjustment process reaches a point $p^* \in int(A(T))$ satisfying $\min z_h(p^*) = \max z_h(p^*)$.

In Case 1, p^* satisfies $p_j^* = 0$ for some $j \in T^-$, which is excluded by Lemma 5, or $p^* \in A(K)$ for some nonempty set $K \subset T$, satisfying $\mid T \backslash K \mid = 1$. In the latter subcase, $p_k^* = v_k$ for some $k \in I^{n+1} \backslash T(c)$. Let $K = T \backslash \{k\}$ if $k \in T^+$, and $K = T \backslash \{-k\}$ if $k \in T^-$, then the price adjustment process continues by tracing a path of points p in $A(K)$, satisfying (5) with T equal to K .

In Case 2, the price adjustment process continues by tracing a path of points p in the set $A(K)$, where $K = T \cup \{+j^*\}$ if $z_{j^*}(p^*) = \max_{h \in I^{n+1}} z_h(p^*)$, and $K = T \cup \{-j^*\}$ if $z_{j^*}(p^*) = \min_{h \in I^{n+1}} z_h(p^*)$, satisfying (5) with T equal to K.

In Case 3, it follows from Lemma 4 that p^* is an equilibrium price vector and the price adjustment process terminates.

In this manner, starting at v, the price adjustment process follows a path of points, corresponding with price vectors, in sets of which the dimension may vary between 1 and n, for an exchange economy with $n+1$ commodities. Such a set $A(T)$ is completely determined by the arbitrarily chosen starting point of the process, $v \in S^n$, and a certain set of positive and negative integers $T \subset I^{n+1} \cup -I^{n+1}$, with $\dim A(T)$ equal to $\mid T \mid -1$. Along the path followed by the process in such a set $A(T)$, the excess demand of any commodity with index $k \in T \cap I^{n+1}$ is kept maximal, and the price of any such commodity may be higher than the initial level. Furthermore, the excess demand of any commodity with index $k \in T \cap -I^{n+1}$ is kept minimal, and the price of such a commodity may be lower than the initial level. The prices of all other commodities in the exchange economy are kept equal to their initial levels, and the excess demands of these commodities are allowed to vary between the levels of minimal and maximal excess demand.

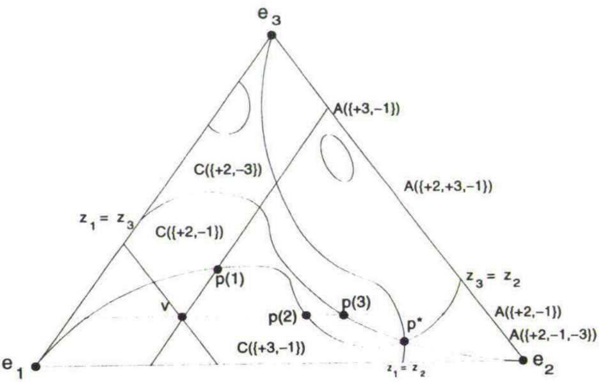

Figure 4: The combination of Figures 2 and 3. The price adjustment process goes from the starting point v via p(1), p(2), p(3) to the equilibrium price vector p*.

If the excess demand of any of the latter commodities becomes equal to either maximal or minimal excess demand as well, then the price adjustment process continues in a set of dimension one higher. On the other hand, as soon as the price of a commodity becomes equal to its initial level again, while having been raised or lowered from its initial value, the price adjustment process continues in a set of one dimension lower. The path of points followed by the price adjustment process, connects the starting point v with a point representing an equilibrium price vector. The price adjustment process terminates at an equilibrium price vector for arbitrary excess demand functions under precisely the conditions which guarantee the existence of at least one equilibrium price vector, while being started at an arbitrary price vector. The informational requirements for this *globally convergent* price adjustment process consist of local information obtained from the excess demand function and global information about the location of the current price vector in relation to the starting point v.

The remainder of this section is devoted to a detailed discussion of the examples in Figures 2 and 3 as depicted in Figure 4. At the starting point v, the excess demand of commodity 3 is maximal and the excess demand of commodity 1 is minimal, since $v \in C(\{+3,-1\})$. The price adjustment process leaves the starting point v by raising the price of commodity 3 and simultaneously lowering the price of the commodity 1 from their respective initial levels v_1 and v_3. Thus, the one-dimensional set $A(\{+3,-1\})$ is entered by the price adjustment process and it traces a curve of points p in S^2 satisfying

$$p_3 \geq v_3 \text{ and } z_3(p) = \max_{h \in I^3} z_h(p)$$

$$p_2 = v_2 \text{ and } z_1(p) \leq z_2(p) \leq z_3(p)$$

$$p_1 \leq v_1 \text{ and } z_1(p) = \min_{h \in I^3} z_h(p),$$

until the point $p(1)$ is reached. At the price vector $p(1)$, it holds that $z_2(p(1))$, the excess demand of commodity 2, is equal to $z_3(p(1))$, the excess demand of commodity 3. Moreover, $z_2(p(1))$ and $z_3(p(1))$ are both maximal. The process continues by raising the price of commodity 2 from its initial level v_2, and by keeping z_2 and z_3 equal to each other. Thus, the set $A(\{+2,+3,-1\})$ is entered and the price adjustment process follows a curve of points p in S^2 satisfying

$$p_j \geq v_j \text{ and } z_j(p) = \max_{h \in I^3} z_h(p) \text{ for } j \in \{2,3\}$$

$$p_1 \leq v_1 \text{ and } z_1(p) = \min_{h \in I^3} z_h(p),$$

until the point $p(2)$ is reached. At the price vector $p(2)$, the price of commodity 3 which had initially been raised, has become equal to its initial level v_3 again. Then, the process continues by keeping the price of commodity 3 at v_3, and by allowing the excess demand of commodity 3, $z_3(p)$, to become less than the maximal excess demand, $z_2(p)$. The latter causes a raise of the price of commodity 2 and simultaneously a lowering of the price of commodity 1. Hence, the process proceeds by entering the one-dimensional set $A(\{+2,-1\})$ and following a curve of points p in S^2 satisfying

$$p_2 \geq v_2 \text{ and } z_2(p) = \max_{h \in I^3} z_h(p)$$

$$p_3 = v_3 \text{ and } z_1(p) \leq z_3(p) \leq z_2(p)$$

$$p_1 \leq v_1 \text{ and } z_1(p) = \min_{h \in I^3} z_h(p),$$

until the point $p(3)$ is reached. At this price vector the excess demand of commodity 3 becomes equal to the minimal excess demand of commodity 1. The process then proceeds by lowering the price of commodity 3 from v_3 and keeping z_1 equal to z_3. Thus, the set $A(\{+2,-1,-3\})$ is entered and a curve of points p in S^2 is followed satisfying

$$p_2 \geq v_2 \text{ and } z_2(p) = \max_{h \in I^3} z_h(p)$$

$$p_j \leq v_j \text{ and } z_j(p) = \min_{h \in I^3} z_h(p) \text{ for } j = 1, 3$$

until the point p^* is reached. At p^*, it holds that $z_1(p^*) = z_2(p^*) = z_3(p^*)$, hence $z(p^*) = 0^3$ by Lemma 4.

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